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# Inclusive Semileptonic $B$ Decays

in

## Soft-Collinear Effective Theory

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# Motivation

$|V_{ub}|$  from  $\bar{B} \rightarrow X_u \ell \bar{\nu}_\ell$ : large background from  $\bar{B} \rightarrow X_c \ell \bar{\nu}_\ell$

$\Rightarrow$  cuts in phase space:  $m_X^2 \sim \Lambda m_b$ ,  $E_X \sim m_b$

- theoretical framework: soft-collinear effective theory (SCET)
- decay rate sensitive to (non-perturbative) **shape function**  
 $\Rightarrow$  hadronic uncertainties in the determination of  $|V_{ub}|$

Possible solutions:

- extract **shape function** from another process and use it as input for  $\bar{B} \rightarrow X_u \ell \bar{\nu}_\ell$
- construct **shape-function independent** relations between decay spectra in  $\bar{B} \rightarrow X_u \ell \bar{\nu}_\ell$  and other processes

most often and successfully used “other“ process:  $\bar{B} \rightarrow X_s \gamma$

# Motivation

$\bar{B} \rightarrow X_c \ell \bar{\nu}_\ell$ : use similar cuts in phase space

particular power counting  $m_c \sim \sqrt{\Lambda m_b}$       ( $1.5 \text{ GeV} \approx \sqrt{0.5 \cdot 4.8} \text{ GeV}$ )

- theoretical framework: soft-collinear effective theory (SCET)
- decay rate sensitive to shape function

- shape function from  $\bar{B} \rightarrow X_c \ell \bar{\nu}_\ell$  as input for  $\bar{B} \rightarrow X_u \ell \bar{\nu}_\ell$
  - shape-function independent relation between  $\bar{B} \rightarrow X_c \ell \bar{\nu}_\ell$  and  $\bar{B} \rightarrow X_u \ell \bar{\nu}_\ell$  decay spectra
- $\Rightarrow$  extraction of  $|V_{ub}|$

$\Rightarrow$  extend SCET to describe massive collinear quarks

$\Rightarrow$  calculate  $\bar{B} \rightarrow X_c \ell \bar{\nu}_\ell$  in SCET

# Outline

## 1. Theory: SCET for massive collinear quarks

- Introduction
- Light-cone kinematics and power counting
- Lagrangian and currents

## 2. Application: $\bar{B} \rightarrow X_c \ell \bar{\nu}_\ell$ in SCET

- subleading order in  $\frac{\Lambda}{m_b}$ , tree level ( $\mathcal{O}(\alpha_s^0)$ )
- leading order in  $\frac{\Lambda}{m_b}$ , with radiative corrections ( $\mathcal{O}(\alpha_s)$ )
  - calculation of the jet function
  - shape-function independent relation between  $\bar{B} \rightarrow X_c \ell \bar{\nu}_\ell$  and  $\bar{B} \rightarrow X_u \ell \bar{\nu}_\ell$  decay spectra

## 3. Conclusions

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Theoretical part:

## Soft-Collinear Effective Theory

### for massive collinear quarks

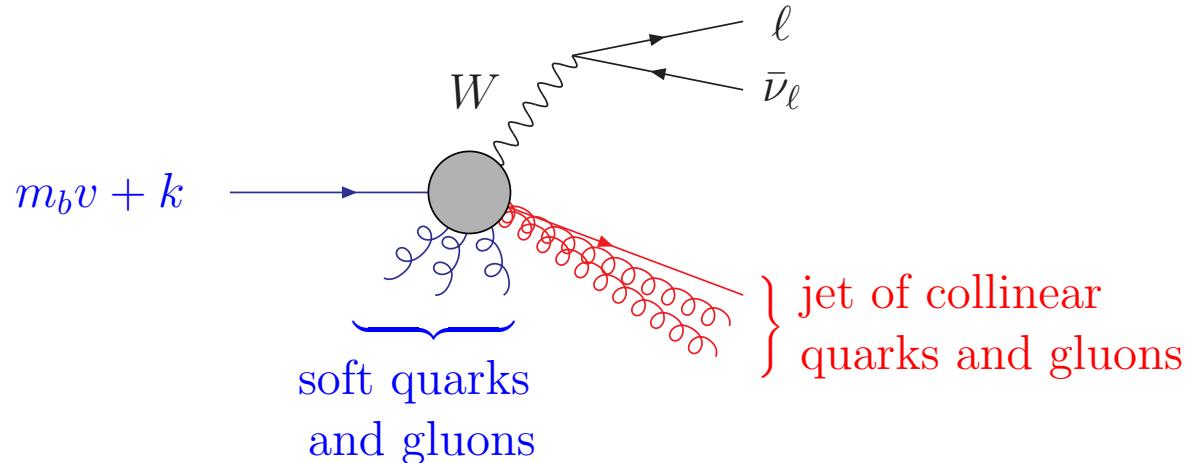
HB, Feldmann, Mannel, Pecjak (2005)

SCET references:

- [Bauer, Fleming, Luke (2000)], [Bauer, Fleming, Pirjol, Stewart (2000)],
  - [Bauer, Stewart (2001)],
  - [Beneke, Chapovsky, Diehl, Feldmann (2002)], [Beneke, Feldmann (2002)]
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# /// $\bar{B} \rightarrow X_c \ell \bar{\nu}_\ell$ in the shape-function region ///

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- massive collinear modes:

$$E_{\text{jet}} = \frac{m_b}{2} + \mathcal{O}(\Lambda) = \mathcal{O}(m_b)$$

$$p_{\text{jet}}^2 - m_c^2 = \mathcal{O}(\Lambda m_b)$$

$$m_c^2 = \mathcal{O}(\Lambda m_b)$$

- soft modes:  $k \sim \mathcal{O}(\Lambda)$ ,  $p_s \sim \mathcal{O}(\Lambda)$

# /// Effective field theories for $\bar{B} \rightarrow X_c \ell \bar{\nu}_\ell$ ///

QCD

all quark and gluon modes propagate

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$$\mu_1 \sim m_b$$

SCET

soft and collinear modes for light quarks and gluons

collinear modes for massive charm quarks ( $m_c \sim \sqrt{\Lambda m_b}$ )

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$$\mu_2 \sim \sqrt{\Lambda m_b}$$

HQET

only soft modes

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$$\mu_3 \sim \Lambda$$

| Factorization :  $d\Gamma \sim \textcolor{green}{H} \cdot \textcolor{red}{J} \otimes \textcolor{blue}{S}$  |

# /// Light-cone kinematics and power counting ///

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light-cone kinematics: light-like vectors  $n_+^\mu$  and  $n_-^\mu$

$$n_+^2 = n_-^2 = 0, \quad n_+ n_- = 2, \quad v \cdot n_\pm \sim 1$$

any Lorentz vector can be written as:

$$p^\mu = (n_+ p) \frac{n_-^\mu}{2} + p_\perp^\mu + (n_- p) \frac{n_+^\mu}{2}, \quad p^2 = (n_+ p)(n_- p) + p_\perp^2$$

power counting:  $\lambda = \sqrt{\frac{p_{\text{jet}}^2}{m_b^2}} = \sqrt{\frac{\Lambda}{m_b}} \ll 1$

$$(n_+ p_c, p_{c\perp}, n_- p_c) \sim m_b(1, \lambda, \lambda^2), \quad p_s \sim m_b(\lambda^2, \lambda^2, \lambda^2)$$

$$m_c \sim \sqrt{\Lambda m_b} \sim m_b \lambda$$

$\Rightarrow$  different fields for collinear and soft modes

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# /// Lagrangian for collinear charm-quark fields ///

interacting with soft and collinear gluons

rewrite QCD Lagrangian:

$$\begin{aligned}\mathcal{L}_{\text{QCD}} &= \bar{\psi}_c (iD - m_c) \psi_c \\ &= \bar{\xi} \left( i n_- D + (iD_\perp - m_c) \frac{1}{i n_+ D} (iD_\perp + m_c) \right) \frac{\not{n}_+}{2} \xi\end{aligned}$$

where  $iD = i\partial + gA$ ,

light-cone projections:

$$\xi(x) = \frac{\not{n}_- \not{n}_+}{4} \psi_c(x) \sim \lambda, \quad \eta(x) = \frac{\not{n}_+ \not{n}_-}{4} \psi_c(x) \sim \lambda^2,$$

$$\psi_c(x) = \xi(x) + \eta(x) = \left( 1 + \frac{1}{i n_+ D} (iD_\perp + m_c) \frac{\not{n}_+}{2} \right) \xi(x)$$

(equation of motion for  $\bar{\eta}$ )



# Multi-pole expansion



$$x^\mu = (n_+ x) \frac{n_-^\mu}{2} + x_\perp^\mu + (n_- x) \frac{n_+^\mu}{2}$$

collinear

soft

$$(n_+ p, \textcolor{red}{p}_\perp, n_- p) \sim (1, \textcolor{red}{\lambda}, \textcolor{blue}{\lambda^2}), \quad p \sim (\textcolor{blue}{\lambda^2}, \textcolor{blue}{\lambda^2}, \textcolor{blue}{\lambda^2})$$

$$(n_- x, \textcolor{red}{x}_\perp, n_+ x) \sim (1, \textcolor{red}{\lambda^{-1}}, \textcolor{blue}{\lambda^{-2}}), \quad x \sim (\textcolor{blue}{\lambda^{-2}}, \textcolor{blue}{\lambda^{-2}}, \textcolor{blue}{\lambda^{-2}})$$

- In transverse and  $n_+$  directions, soft fields have larger wavelengths than collinear ones, i.e. vary more slowly

⇒ expand position arguments of soft fields in those directions

$$A_s(x) \rightarrow A_s(\textcolor{blue}{x}_-) + [x_\perp \partial_\perp A_s](\textcolor{blue}{x}_-) + \dots$$

$$\text{with} \quad x_- = (\textcolor{blue}{n_+ x}) \frac{n_-}{2}$$



# Expand Lagrangian in $\lambda$



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$$\mathcal{L}_{\text{QCD}} = \bar{\xi} \left( \textcolor{blue}{in_- D} + (\textcolor{red}{iD_\perp} - m_c) \frac{1}{\textcolor{green}{in_+ D}} (\textcolor{red}{iD_\perp} + m_c) \right) \frac{\not{n}_+}{2} \xi$$

split covariant derivative

$$\textcolor{red}{iD_\perp} = \textcolor{red}{iD_{\perp c}} + \textcolor{blue}{gA_{\perp s}}, \quad \textcolor{green}{in_+ D} = \textcolor{green}{in_+ D_c} + \textcolor{blue}{gn_+ A_s}$$

and

$$\text{multi-pole expansion } A_s(x) = A_s(\textcolor{blue}{x_-}) + \dots$$

leading-order Lagrangian:

$$\mathcal{L}_\xi^{(0)} = \bar{\xi} \left( \textcolor{blue}{in_- D} + (\textcolor{red}{iD_{\perp c}} - m_c) \frac{1}{\textcolor{green}{in_+ D_c}} (\textcolor{red}{iD_{\perp c}} + m_c) \right) \frac{\not{n}_+}{2} \xi$$



# Expand Lagrangian in $\lambda$



$$\mathcal{L}_{\text{QCD}} = \bar{\xi} \left( \textcolor{blue}{in_- D} + (\textcolor{red}{iD_\perp} - m_c) \frac{1}{\textcolor{green}{in_+ D}} (\textcolor{red}{iD_\perp} + m_c) \right) \frac{\not{n}_+}{2} \xi$$

$$iD_\perp = \textcolor{red}{iD_{\perp c}} + gA_{\perp s}, \quad \textcolor{blue}{in_+ D} = \textcolor{green}{in_+ D_c} + gn_+ A_s, \quad A_s(x) = A_s(\textcolor{blue}{x_-}) + \dots$$

power-suppressed terms proportional to  $m_c$ :

$$\mathcal{L}_{\xi m}^{(1)} = \textcolor{red}{m_c} \bar{\xi} [\textcolor{blue}{gA_{\perp s}}, \frac{1}{\textcolor{green}{in_+ D_c}}] \frac{\not{n}_+}{2} \xi$$

$$\mathcal{L}_{\xi m1}^{(2)} = \textcolor{red}{m_c^2} \bar{\xi} \frac{1}{\textcolor{green}{in_+ D_c}} gn_+ A_s \frac{1}{\textcolor{green}{in_+ D_c}} \frac{\not{n}_+}{2} \xi$$

$$\mathcal{L}_{\xi m2}^{(2)} = \textcolor{red}{m_c} \bar{\xi} [\frac{1}{\textcolor{green}{in_+ D_c}} gn_+ A_s \frac{1}{\textcolor{green}{in_+ D_c}}, \textcolor{red}{iD_{\perp c}}] \frac{\not{n}_+}{2} \xi$$

$$\mathcal{L}_{\xi m3}^{(2)} = \textcolor{red}{m_c} \bar{\xi} [(\textcolor{brown}{x_\perp} \partial_\perp gA_{\perp s}), \frac{1}{\textcolor{green}{in_+ D_c}}] \frac{\not{n}_+}{2} \xi$$



# SCET currents



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$$(\bar{\psi}_c \Gamma b)_{\text{QCD}} \rightarrow e^{-im_b v \cdot x} \left[ J^{(0)} + J^{(1)} + J^{(2)} + \dots \right]$$

tree level:

$$J^{(0)} = \bar{\xi} \Gamma W_c h_v,$$

$$J_{\textcolor{red}{m}}^{(1)} = \bar{\xi} (i \overleftarrow{D}_{\perp c} + \textcolor{red}{m}_c) \frac{1}{-in_+ \overleftarrow{D}_c} \frac{\not{n}_+}{2} \Gamma W_c h_v$$

$$J_{\textcolor{red}{m}1}^{(2)} = \bar{\xi} (i \overleftarrow{D}_{\perp c} + \textcolor{red}{m}_c) \frac{1}{-in_+ \overleftarrow{D}_c} \frac{\not{n}_+}{2} \Gamma W_c [x_\perp D_{\perp s} h_v]$$

$$J_{\textcolor{red}{m}2}^{(2)} = \bar{\xi} (i \overleftarrow{D}_{\perp c} + \textcolor{red}{m}_c) \frac{1}{in_+ \overleftarrow{D}_c} \frac{\not{n}_+}{2} \Gamma \frac{\not{n}_-}{2m_b} [i \overleftarrow{D}_{\perp c} W_c] h_v$$

with a collinear Wilson line :  $W_c \equiv P \exp \left[ ig \int_{-\infty}^0 ds n_+ A_{\textcolor{green}{c}}(x + sn_+) \right]$

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Application:

$$\bar{B} \rightarrow X_c \ell \bar{\nu}_\ell$$

in

**Soft-Collinear Effective Theory**

HB, Feldmann, Mannel, Pecjak (2005)

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# Our goal

- shape function from  $\bar{B} \rightarrow X_c \ell \bar{\nu}_\ell$  as input for  $\bar{B} \rightarrow X_u \ell \bar{\nu}_\ell$
  - shape-function independent relation between  $\bar{B} \rightarrow X_c \ell \bar{\nu}_\ell$  and  $\bar{B} \rightarrow X_u \ell \bar{\nu}_\ell$  decay spectra
- ⇒ extraction of  $|V_{ub}|$

⇒ calculate  $\bar{B} \rightarrow X_c \ell \bar{\nu}_\ell$  in SCET

- subleading order in  $\frac{\Lambda}{m_b}$ , tree level ( $\mathcal{O}(\alpha_s^0)$ )
- leading order in  $\frac{\Lambda}{m_b}$ , with radiative corrections ( $\mathcal{O}(\alpha_s)$ )

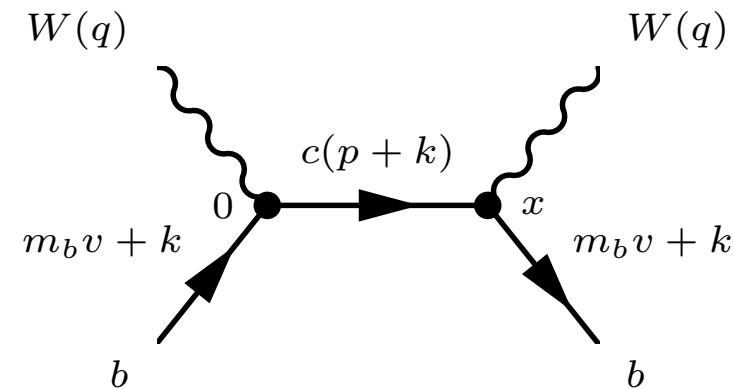
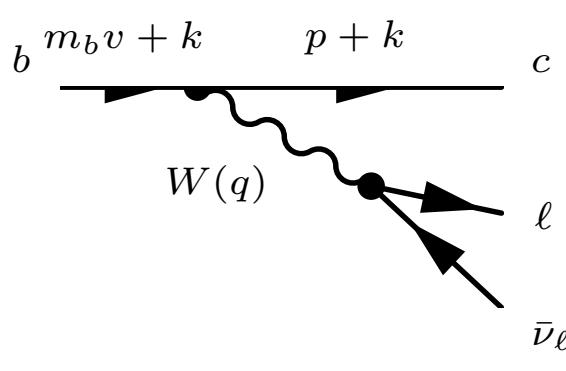


# Decay rate and hadronic tensor



$$d\Gamma \sim \frac{G_F^2 |V_{cb}|^2}{\pi^3} L_{\mu\nu} W^{\mu\nu}$$

$$W^{\mu\nu} = \frac{1}{\pi} \text{Im} \langle \bar{B}(v) | T^{\mu\nu} | \bar{B}(v) \rangle \quad \text{with} \quad T^{\mu\nu} = i \int d^4x e^{-iq \cdot x} \text{T}\{ J^{\dagger\mu}(x) J^\nu(0) \}$$



collinear quark propagator :  $S_c(p + k) = \frac{\not{p}_-}{2} \frac{i}{n_-(p + k) - \frac{\not{m}_c^2}{n_+ p}} = \frac{\not{p}_-}{2} \frac{i}{u + n_- k}$

consider particular kinematic variable:  $u = n_- p - \frac{\not{m}_c^2}{n_+ p}$

# /// Subleading order in $\frac{\Lambda}{m_b}$ , tree level ///

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Factorization  $d\Gamma \sim \textcolor{green}{H} \cdot \textcolor{red}{J} \otimes \textcolor{blue}{S}$  to  $\mathcal{O}\left(\frac{\Lambda}{m_b}\right)$  ✓

- $\textcolor{green}{H} = 1$
- $\textcolor{red}{J}^{(0)} = \delta(u - \omega)$
- leading shape function  $\textcolor{blue}{S}(\omega)$ ,  $\textcolor{blue}{S}_{\text{part}}^{(0)}(\omega) = \delta(\omega + n_- k)$   
several subleading shape functions



# Decay spectra



$\bar{B} \rightarrow X_u \ell \bar{\nu}_\ell$ :

Bosch, Neubert, Paz

$$\begin{aligned} \frac{1}{\Gamma_u} \frac{d\Gamma}{d(n-p)} &= \left(1 - \frac{14}{3} \frac{n-p}{m_b}\right) S(n-p) + \frac{s(n-p)}{2m_b} \\ &\quad + \frac{1}{3m_b} [t(n-p) + u_a(n-p) - 5u_s(n-p)] \end{aligned}$$

$\bar{B} \rightarrow X_c \ell \bar{\nu}_\ell$  ( $m_c \sim \sqrt{\Lambda m_b} \sim m_b \lambda$ ):

HB, Feldmann, Mannel, Pecjak

$$\begin{aligned} \frac{1}{\Gamma_c} \frac{d\Gamma}{du} &= \left(1 - \frac{14}{3} \frac{u}{m_b} - 8 \frac{m_c^2}{m_b^2}\right) S(u) + \frac{s(u)}{2m_b} - 4 \frac{m_c^2}{m_b^2} t_1(u) \\ &\quad + \frac{1}{3m_b} [t(u) + u_a(u) - 5u_s(u)] \end{aligned}$$

$$\text{with } u = n-p - \frac{m_c^2}{n+p}, \quad \Gamma_{u,c} = \frac{G_F^2 m_b^5 |V_{(u,c)b}|^2}{192\pi^3}$$

# /// With radiative corrections ( $\mathcal{O}(\alpha_s)$ ) ///

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leading order in  $\frac{\Lambda}{m_b}$

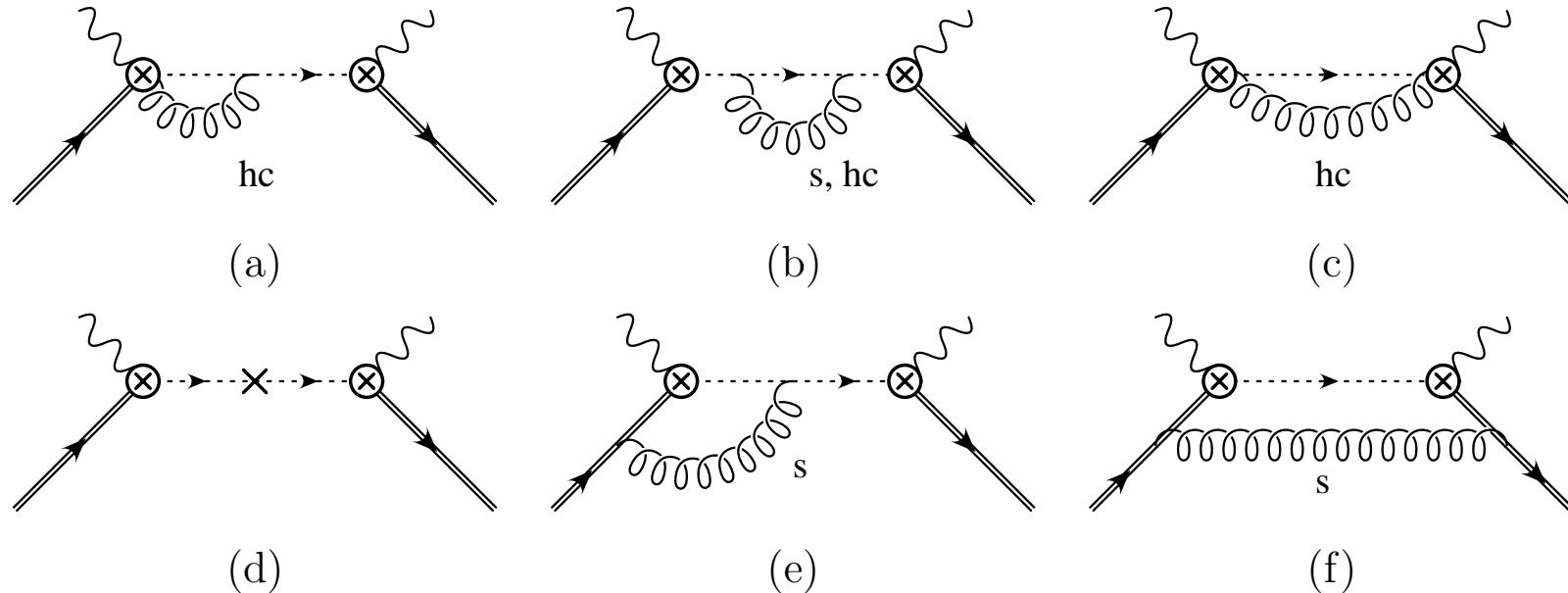
Factorization  $d\Gamma \sim \textcolor{green}{H} \cdot \textcolor{red}{J} \otimes \textcolor{blue}{S}$  to one-loop  $\alpha_s$ -accuracy ✓

- $\textcolor{green}{H} = 1 + \alpha_s \cdots$
- $\textcolor{red}{J} = \delta(u - \omega) + \alpha_s \cdots$
- leading shape function  $\textcolor{blue}{S}(\omega)$  only

$$\begin{aligned}\textcolor{blue}{S}_{\text{part}}(\omega) &= \textcolor{blue}{S}_{\text{part}}^{(0)}(\omega) + \textcolor{blue}{S}_{\text{part}}^{(1)}(\omega) \\ &= \delta(\omega + n_- k) + \alpha_s \cdots\end{aligned}$$

# /// $\bar{B} \rightarrow X_c \ell \bar{\nu}_\ell$ in SCET at one loop ///

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collinear gluon-exchange + mass counterterm  $\Rightarrow J$

$H, S$  : the same as in  $\bar{B} \rightarrow X_u \ell \bar{\nu}_\ell$

Bosch, Lange, Neubert, Paz

# /// Calculation of one-loop jet function ///

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$$D_{hc}^{(1)} = \mathcal{J}_{hc}^{(1)} \left[ \bar{h}_v \bar{\Gamma}_j^\mu \frac{\eta_-}{2} \Gamma_i^\nu h_v \right]$$

where

$$\begin{aligned} \mathcal{J}_{hc}^{(1)} &= \frac{C_F \alpha_s}{4\pi} \frac{i}{u'} \left\{ \frac{4}{\epsilon^2} + \frac{3}{\epsilon} - \frac{4}{\epsilon} \ln \left( \frac{-n_+ p u'}{\mu^2} \right) \right. \\ &+ 7 - \frac{\pi^2}{3} - 3 \ln \left( \frac{-n_+ p u'}{\mu^2} \right) + 2 \ln^2 \left( \frac{-n_+ p u'}{\mu^2} \right) \\ &+ \frac{2\pi^2}{3} - 4 \text{Li}_2 \left( 1 + \frac{m_p}{u'} \right) \\ &\left. + \frac{m_p}{m_p + u'} - \frac{m_p (m_p + 2u')}{(m_p + u')^2} \ln \left( -\frac{u'}{m_p} \right) \right\} \end{aligned}$$

$$\text{with } u = n_- p - \frac{m_c^2}{n_+ p}, \quad u' = u + n_- k, \quad \alpha_s \equiv \alpha_s(\mu), \quad m_p \equiv \frac{m_c^2}{n_+ p}$$


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# The star distributions



$$\int_{\leq 0}^M dx F(x) \left( \frac{1}{x} \right)_*^{[m]} = \int_0^M dx \frac{F(x) - F(0)}{x} + F(0) \ln \left( \frac{M}{m} \right)$$

$$\int_{\leq 0}^M dx F(x) \left( \frac{\ln(x/m)}{x} \right)_*^{[m]} = \int_0^M dx \frac{F(x) - F(0)}{x} \ln \frac{x}{m} + \frac{F(0)}{2} \ln^2 \left( \frac{M}{m} \right)$$

$$-\frac{1}{\pi} \text{Im} \left[ \ln \left( -\frac{u'}{m} \right) \frac{1}{u'} \right] = \left( \frac{1}{u'} \right)_*^{[m]}$$

$$-\frac{1}{\pi} \text{Im} \left[ \ln^2 \left( -\frac{u'}{m} \right) \frac{1}{u'} \right] = 2 \left( \frac{\ln(u'/m)}{u'} \right)_*^{[m]} - \frac{\pi^2}{3} \delta(u')$$

$$-\frac{1}{\pi} \text{Im} \left[ \frac{1}{u'} \left[ \text{Li}_2 \left( 1 + \frac{m}{u'} \right) \right] \right] = - \left( \frac{\ln(u'/m)}{u'} \right)_*^{[m]} + \frac{1}{u'} \ln \left( 1 + \frac{u'}{m} \right) \theta(u')$$

$$u' \equiv u' + i\epsilon$$



# One-loop jet function



$$\textcolor{red}{J}^{(1)} \otimes S_{\text{part}}^{(0)} = \textcolor{red}{J}^{(1)} \otimes \delta(\omega + n_- k) = J^{(1)}(u', n_+ p) = \frac{1}{\pi} \text{Im} \left[ i \mathcal{J}_{\text{hc,finite}}^{(1)} \right]$$

$$\begin{aligned} J^{(1)}(u', n_+ p) &= \frac{C_F \alpha_s}{4\pi} \left\{ \right. \\ &\quad \left( 7 - \pi^2 \right) \delta(u') - 3 \left( \frac{1}{u'} \right)_*^{[\mu^2 / n_+ p]} + 4 \left( \frac{\ln(u' n_+ p / \mu^2)}{u'} \right)_*^{[\mu^2 / n_+ p]} \\ &\quad + \left( \frac{u'}{(m_p + u')^2} - \frac{4}{u'} \ln \left( 1 + \frac{u'}{m_p} \right) \right) \theta(u') + \left( 1 + \frac{2\pi^2}{3} \right) \delta(u') \\ &\quad \left. - \left( \frac{1}{u'} \right)_*^{[m_p]} + 4 \left( \frac{\ln(u' / m_p)}{u'} \right)_*^{[m_p]} \right\} \end{aligned}$$

$$J(u', n_+ p) = \delta(u') + J^{(1)}(u', n_+ p)$$

$$\textcolor{red}{J} = \delta(u - \omega) + J^{(1)}(u - \omega, n_+ p)$$

# /// The partially integrated $U$ spectrum ///

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consider hadronic variables

$(U$  is mass-scheme dependent)

$$U = \textcolor{blue}{u} + \bar{\Lambda} = n_- P - \frac{\textcolor{red}{m}_c^2}{n_+ p} \quad \longleftrightarrow \quad \textcolor{brown}{P}_+ = n_- P \quad (b \rightarrow u)$$

partially integrated spectrum: (cut-off parameter  $\Delta \approx 0.6$  GeV)

$$F_c(\Delta) = \frac{1}{\Gamma_c} \int_0^\Delta dU \frac{d\Gamma_c}{dU} = \frac{\Gamma_c(U < \Delta)}{\Gamma_c} = \textcolor{brown}{F}_{\textcolor{blue}{u}}(\Delta) + \textcolor{red}{F}_{\textcolor{red}{m}}(\Delta)$$

theoretical result contains shape function  $\hat{S}$ :

$$\textcolor{brown}{F}_{\textcolor{blue}{u}}(\Delta) \sim \int_0^\Delta d\hat{\omega} \hat{S}(\hat{\omega}, \mu_i) f_u \left( \frac{m_b(\Delta - \hat{\omega})}{\mu_i^2} \right)$$

$$\textcolor{red}{F}_{\textcolor{red}{m}}(\Delta) \sim \int_0^\Delta d\hat{\omega} \hat{S}(\hat{\omega}, \mu_i) \textcolor{red}{f}_m \left( \frac{m_b(\Delta - \hat{\omega})}{m_c^2} \right)$$

# /// Relating $b \rightarrow c$ and $b \rightarrow u$ decays ///

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shape-function independent relation  $\implies$  construct weight function

$$\begin{aligned}
 F_u(\Delta) &= \int_0^\Delta dP_+ \underbrace{\frac{d\Gamma_u}{dP_+}}_{\text{exp.}} = \frac{\Gamma_u}{\Gamma_c} \int_0^\Delta dU W(\Delta, U) \frac{d\Gamma_c}{dU} \\
 &\approx \frac{|V_{ub}|^2}{|V_{cb}|^2} \int_0^\Delta dU \underbrace{W(\Delta, U)}_{\text{theory}} \underbrace{\frac{d\Gamma_c}{dU}}_{\text{exp.}}
 \end{aligned} \tag{1}$$

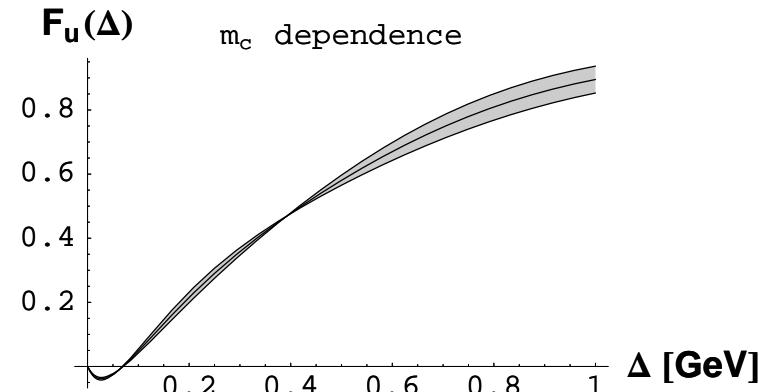
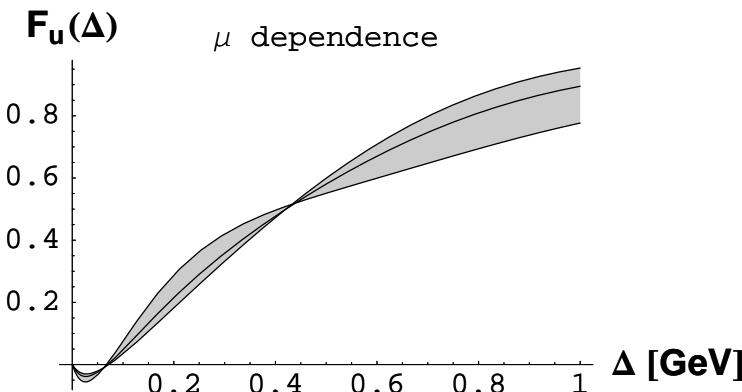
$$W(\Delta, U) = 1 - f_m \left( \frac{m_b(\Delta - U)}{m_c^2} \right) + \mathcal{O}(\alpha_s^2) + \mathcal{O}\left(\frac{\Lambda}{m_b}\right)$$

$$\left\{ \begin{array}{c} \frac{d\Gamma_c}{dU} \\ W \end{array} \right\} \text{mass-scheme dependent} \longleftrightarrow (1) \text{ mass-scheme independent}$$


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# /// Perturbative uncertainties of weight function ///

$F_u(\Delta)$  from weight function and theoretical  $U$  spectrum ( $b \rightarrow c$ )



factorization-scale dependence ( $1 \text{ GeV} < \mu_i < 2.25 \text{ GeV}$ )  $\sim 10\text{-}15\%$

charm-mass dependence ( $m_c \pm 0.15 \text{ GeV}$ )  $< 10\%$

- could be resolved by calculating  $\alpha_s^2$  corrections to  $J$

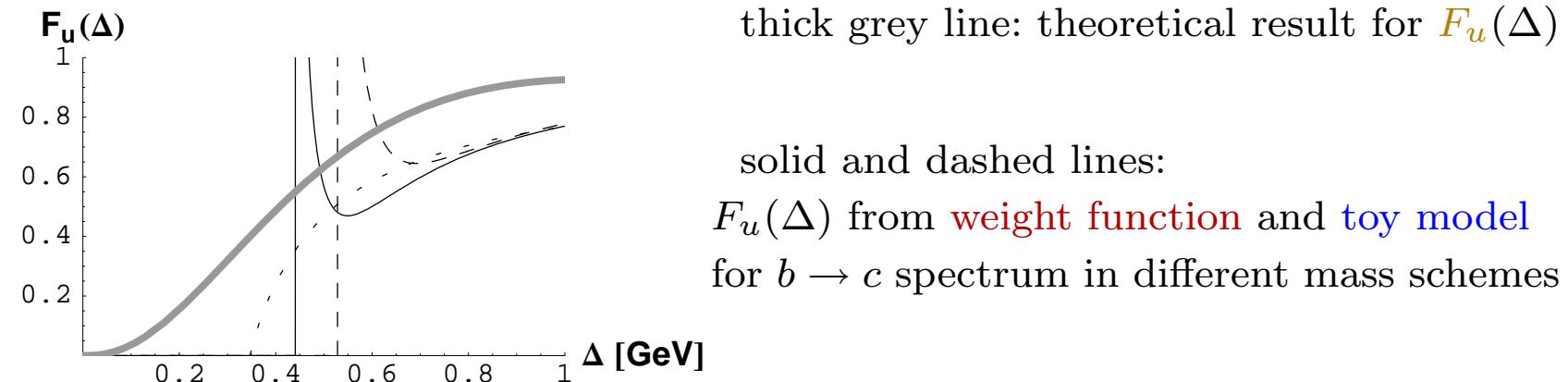
but:  $\frac{\Lambda}{m_b}$  corrections  $\Rightarrow$  systematic uncertainties  $\gtrsim 20\%$



# Illustrate resonance effects



$D$  and  $D^*$  resonance  $\Rightarrow$  toy model  $\frac{1}{\Gamma_c} \frac{d^2\Gamma_c}{d(n_- P) d(n_+ P)} \sim \delta(P^2 - \overline{M}_D^2)$



- larger values of  $\Delta$ : resonance structure washed out,  
results in different mass schemes converge  
 $\Rightarrow$  cut-off parameter  $\Delta$  sufficiently large to avoid sensitivity  
to the shape of the spectrum in the resonance region

# Conclusions

- SCET: describe processes involving both **collinear** and **soft** modes
- SCET for **massive collinear** quarks ( $m_c \sim \sqrt{\Lambda m_b} \sim m_b \lambda$ ):  
Lagrangian and currents

Application:  $\bar{B} \rightarrow X_c \ell \bar{\nu}_\ell$  in SCET

- Factorization:  $d\Gamma \sim \textcolor{green}{H} \cdot \textcolor{red}{J} \otimes \textcolor{blue}{S}$
- particular kinematic variable

$$\textcolor{blue}{U} = n_- P - \frac{\textcolor{red}{m}_c^2}{n_+ p} \quad \longleftrightarrow \quad \textcolor{brown}{P}_+ = n_- P \quad (b \rightarrow u)$$



# Conclusions



## 1. subleading order in $\frac{\Lambda}{m_b}$ , tree level: Factorization ✓

- compare with  $\bar{B} \rightarrow X_u \ell \bar{\nu}_\ell$ : ✓  
new effective shape function  $t_1(u)$

leading shape function from  $\bar{B} \rightarrow X_c \ell \bar{\nu}_\ell$  as input  
for  $\bar{B} \rightarrow X_u \ell \bar{\nu}_\ell \Rightarrow$  determination of  $|V_{ub}/V_{cb}|$

## 2. $\mathcal{O}(\alpha_s)$ , leading order in $\frac{\Lambda}{m_b}$ :

- Factorization:  $d\Gamma \sim H \cdot J \otimes S$  ✓
- calculation of  $J$   
shape-function independent relation between partially integrated  $\bar{B} \rightarrow X_c \ell \bar{\nu}_\ell$  and  $\bar{B} \rightarrow X_u \ell \bar{\nu}_\ell$  decay spectra  
 $\Rightarrow$  determination and crosscheck of  $|V_{ub}/V_{cb}|$
- wait for experimental information on  $U$  spectrum in  $\bar{B} \rightarrow X_c \ell \bar{\nu}_\ell$